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A contribution to the discussion of the matter–antimatter asymmetry problem

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Abstract

Using a more accurate effective Hamiltonian governig the time evolution in the particle–antiparticle subspace of states than the one obtained within the Lee–Oehme–Yang approach we show that in the case of particles created at the instant $t = t_0$ only the masses of a stable particle and its antiparticle are the same at all $t \geq t_0$ in a CPT invariant system, whereas the masses of an unstable particle and its antiparticle are equal only at $t = t_0$ and then during their time evolution they become slightly different for times $t \gg t_0$ if CP symmetry is violated but CPT symmetry holds. This property is used to show that if the baryon number B is not conserved then the asymmetry between numbers of unstable baryons and antibaryons can arise in a CPT invariant system at $t \gg t_0$ even in the thermal equilibrium state of this system.

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1 Introduction.

The knowledge of all the subtleties of the difference between a particle and its antiparticle has a fundamental meaning for understanding our Universe, and especially for the explanation of the problem why the observed Universe contains (acording to the physical and astrophysical data) an excess of matter over antimatter. Searching for properties of the unstable particle–antiparticle pairs one usually uses an effective nonhermitean Hamiltonian [1] — [14], say $H_{\parallel}$, which in general can depend on time t [10],

$$H_{\parallel} \equiv M - \frac{i}{2}\Gamma, \tag{1}$$

where

$$M = M^{+}, \;\; \Gamma = \Gamma^{+}, \tag{2}$$

are (2×2) matrices, acting in a two–dimensional subspace $\mathcal{H}_{\parallel}$ of the total state space $\mathcal{H}$ and M is called the mass matrix, Γ is the decay matrix [1] — [4]. The standard method of derivation of such $H_{\parallel}$ bases on a modification of the Weisskopf–Wigner (WW) approximation [15]. Lee, Oehme and Yang (LOY) have adapted WW aproach to the case of a two particle subsystem [1] — [3] to obtain their effective Hamiltonian $H_{\parallel} \equiv H_{LOY}$. Almost all properties of the neutral kaon complex, or another particle–antiparticle subsystem can be described by solving the Schrödinger–like evolution equation [1] — [14], [16] — [21] (we use $\hbar = c = k = 1$ units)

$$i\frac{\partial}{\partial t}|\psi; t >_{\parallel} = H_{\parallel}|\psi; t >_{\parallel} \tag{3}$$

with the initial condition [21, 17]

$$\| \, |\psi; t = t_0 >_{\parallel} \| = 1, \;\; |\psi; t < t_0 >_{\parallel} = 0, \tag{4}$$

for $|\psi; t >_{\parallel}$ belonging to the subspace $\mathcal{H}_{\parallel} \subset \mathcal{H}$ spanned, e.g., by orthonormal neutral kaons states $|K_0 >$, $|\overline{K}_0 >$, and so on, (then states corresponding with the decay products belong to $\mathcal{H} \ominus \mathcal{H}_{\parallel} \stackrel{\text{def}}{=} \mathcal{H}_{\perp}$).

Solutions of Eq. (3) can be written in matrix form and such a matrix defines the evolution operator (which is usually nonunitary) $U_{\parallel}(t)$ acting in $\mathcal{H}_{\parallel}$:

$$|\psi; t >_{\parallel} = U_{\parallel}(t)|\psi; t_0 = 0 >_{\parallel} \stackrel{\text{def}}{=} U_{\parallel}(t)|\psi >_{\parallel}, \tag{5}$$

where,

$$|\psi>_{\parallel} \equiv a_1|\mathbf{1}> + a_2|\mathbf{2}>, \tag{6}$$

and $|\mathbf{1}>$ stands for vectors of the $|K_0>$, $|B_0>$, etc., type and $|\mathbf{2}>$ denotes antiparticles of the particle "1": $|\overline{K}_0>$, $\overline{B}_0>$, and so on, $<\mathbf{j}|\mathbf{k}>= \delta_{jk}$, $j,k = 1,2$. In many papers it is assumed that the real parts, $\Re(.)$, of diagonal matrix elements of $H_{\parallel}$:

$$\Re\,(h_{jj}) \equiv M_{jj}, \quad (j = 1,2), \tag{7}$$

where

$$h_{jk} = <\mathbf{j}|H_{\parallel}|\mathbf{k}>, \ (j,k = 1,2), \tag{8}$$

correspond to the masses of particle "1" and its antiparticle "2" respectivelly [1] — [8] (and such an interpretation of $\Re\,(h_{11})$ and $\Re\,(h_{22})$ will be used in this paper), whereas imaginary parts, $\Im(.)$,

$$-2\Im\,(h_{jj}) \equiv \Gamma_{jj}, \quad (j = 1,2), \tag{9}$$

are interpreted as the decay widths of these particles [1] — [8].

Physicists look for the mechanism generating the observed baryon–antibaryon asymmetry. Such a mechanism is necessary for an explanation of the matter–antimatter asymmetry in the Universe. Most theories of dynamic generation of the baryon assymetry use the so–called Sakharov conditions [23] — [26]. These conditions do not require CPT symmetry to be violated. The existence of C and CP violating processes, the violation of the baryon number and the existence of nonequilibrium processes are sufficient for baryogenesis [23] — [26]. These conditions can be met in many papers describing different ways of generating the baryon asymmetry. Unfortunately, there is no answer to the question which one of them, if any, is corret. Maybe for this reason some models of processes producing matter–antimatter masses difference, in which a small violation of CPT symmetry at the origin is assumed, are also considered [26] — [30]. Such models are discussed in spite of the fact that CPT symmetry is a fundamental theorem, called the CPT Theorem, of axiomatic quantum field theory which follows from locality, Lorentz invariance, and unitarity [31]. This illustrates the importance of the problem of finding the mechanism producing the matter–antimatter mass asymmetry.

One consequence of the CPT theorem is that under the product of operations $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$ the total Hamiltonian H of the system considered must be

invariant. From this property and from the properties of the LOY theory of time evolution in the subspace of states of two particle subsystem prepared at some initial instant t_0 and then evolving in time $t > t_0$ [1], one usually infers that particles and antiparticles have exactly the same mass. Properties of singularities of scattering amplitudes appearing in S–matrix theory provide one with reasons for a similar conclusion. Generally, such a conlusion is considered to be the obvious.

The aim of this paper is to show by using a more accurate approximation than the LOY theory, that only the masses of a stable particle and its antiparticle are equal in the CPT–invariant system whereas the masses of an unstable particle and its simultaneously created at $t = t_0$ antiparticle must be slightly different at $t \gg t_0$ when CPT symmetry holds but CP symmetry does not. This effect is found by analysing only the properties of approximate solutions of the Schrödinger equation for the initial conditions of type (4), (6). The paper is organized as follows. Formulae appearing in the LOY approach and the derivation of the more accurate effective Hamiltonian $H_{\parallel}$ than H_{LOY} are described in short in Sec. 2. The properties of matrix elements of the $H_{\parallel}$ implied by CPT symmetry are discussed in Sec. 3. Possibilities of an experimental verification of the results obtained in Sec. 3 are considered in Sec. 4. Sec. 5 contains a general discussion of the relations obtained in Sec. 3., and also some attempts to show that the mechanism following from the final relation of Sec. 3 can generate a conribution to the observed matter–antimatter asymmetry. A summary and final remarks can be found in Sec. 6.

2 Preliminaries.

2.1 H_{LOY} and CPT–symmetry.

Now, let us consider briefly some properties of the LOY model. Let H be the total (selfadjoint) Hamiltonian, acting in $\mathcal{H}$ — then the total unitary evolution operator$U(t)$ fulfills the Schrödinger equation

$$i\frac{\partial}{\partial t}U(t)|\phi >= HU(t)|\phi >, \;\; U(0) = I, \tag{10}$$

where I is the unit operator in $\mathcal{H}$, $|\phi >\equiv |\phi; t_0 = 0 >\in \mathcal{H}$ is the initial state of the system:

$$|\phi >\equiv |\psi >_{\parallel}, \tag{11}$$

$t \geq t_0 = 0$, and, in our case $|\phi; t >= U(t)|\phi >$. Let P denote the projection operator onto the subspace $\mathcal{H}_{\parallel}$:

$$P\mathcal{H} = \mathcal{H}_{\parallel}, \quad P = P^2 = P^+, \tag{12}$$

then the subspace of decay products $\mathcal{H}_{\perp}$ equals

$$\mathcal{H}_{\perp} = (I - P)\mathcal{H} \stackrel{\text{def}}{=} Q\mathcal{H}, \quad Q \equiv I - P. \tag{13}$$

For the case of neutral kaons or neutral B–mesons, etc., the projector P can be chosen as follows:

$$P \equiv |\mathbf{1} >< \mathbf{1}| + |\mathbf{2} >< \mathbf{2}|. \tag{14}$$

We assume that the time independent basis vectors $|K_0 >$ and $|\overline{K}_0 >$ are defined analogously to the corresponding vectors used in the LOY theory of time evolution in neutral kaon complex [1]: Vectors $|K_0 >$ and $|\overline{K}_0 >$ can be identified with the eigenvectors of the so–called free Hamiltonian $H^{(0)} = H - H_I$, where H_I denotes the interactions which are responsible for transitions between eigenvectors of $H^{(0)}$, i.e., for the decay process.

In the LOY approach it is assumed that vectors $|\mathbf{1} >$, $|\mathbf{2} >$ considered above are the eigenstates of $H^{(0)}$ for a 2-fold degenerate eigenvalue m_0:

$$H^{(0)}|\mathbf{j} >= m_0|\mathbf{j} >, \quad j = 1, 2. \tag{15}$$

This means that

$$[P, H^{(0)}] = 0. \tag{16}$$

The condition guaranteeing the occurence of transitions between subspaces $\mathcal{H}_{\parallel}$ and $\mathcal{H}_{\perp}$, i.e., a decay process of states in $\mathcal{H}_{\parallel}$, can be written as follows

$$[P, H_I] \neq 0. \tag{17}$$

Usually, in LOY and related approaches, it is assumed that

$$\Theta H^{(0)}\Theta^{-1} = {H^{(0)}}^{+} \equiv H^{(0)}, \tag{18}$$

where Θ is the antiunitary operator:

$$\Theta \stackrel{\text{def}}{=} \mathcal{CPT}. \tag{19}$$

Relation (18) is a particular form of the general transformation rule [31] — [34]: $\Theta \mathcal{O} \Theta^{-1} \stackrel{\text{def}}{=} \mathcal{O}_{CPT}^{+}$, where $\mathcal{O}$ is an arbitrary linear operator. Basic properties of anti–linear and linear operators, their products and commutators are described, eg., in [5, 31, 32, 33].

The subspace of neutral kaons $\mathcal{H}_{\|}$ is assumed to be invariant under Θ:

$$\Theta P \Theta^{-1} = P. \tag{20}$$

In the kaon rest frame, the time evolution for $t \geq t_0 = 0$ is governed by the Schrödinger equation (10) with the initial condition (4), where the initial state of the system has the form (11), (6). Within assumptions (15) — (17) and assuming the following form of $|\psi; t>_{\|}$ for $t \geq t_0$ (see [1], formula (21)),

$$|\psi; t>_{\|} = e^{-\frac{\lambda}{2}t} |\psi>_{\|}, \tag{21}$$

(where λ is a complex number, $\Re\,(\lambda) > 0$), the Weisskopf–Wigner approach leads to the following formulae for the matrix elements h_{jk}^{LOY} of H_{LOY} (see, e.g., [1, 2, 7, 4, 35]):

$$\begin{aligned} h_{jk}^{LOY} &= \, <\mathbf{j}|H_{LOY}|\mathbf{k}> = H_{jk} - \Sigma_{jk}(m_0) && (22) \\ &= M_{jk}^{LOY} - \frac{i}{2}\Gamma_{jk}^{LOY}, \quad (j,k = 1,2), && (23) \end{aligned}$$

where, in this case,

$$\begin{aligned} H_{jk} &\equiv \, <\mathbf{j}|PHP|\mathbf{k}> = <\mathbf{j}|H|\mathbf{k}> \\ &\equiv \, <\mathbf{j}|(H^{(0)} + H_I)|\mathbf{k}> \equiv m_0 \delta_{jk} + <\mathbf{j}|H_I|\mathbf{k}>, \end{aligned} \tag{24}$$

$\Sigma_{jk}(\epsilon) = <\mathbf{j} \mid \Sigma(\epsilon) \mid \mathbf{k}>$ and

$$\Sigma(\epsilon) = PHQ \frac{1}{QHQ - \epsilon - i0} QHP \stackrel{\text{def}}{=} \Sigma^R(\epsilon) + i\Sigma^I(\epsilon), \tag{25}$$

$$\begin{aligned} \Sigma^R(\epsilon) &= PHQ\,\mathbf{P}\frac{1}{QHQ - \epsilon} QHP, && (26) \\ \Sigma^I(\epsilon) &= \pi PHQ \delta(QHQ - \epsilon) QHP, && (27) \end{aligned}$$

and for real ϵ, $\Sigma^R(\epsilon) = \Sigma^R(\epsilon)^+$ and $\Sigma^I(\epsilon) = \Sigma^I(\epsilon)^+$. (In Eq. (26) **P** denotes principal value).

Now, if $\Theta H_I \Theta^{-1} = H_I$, then using, e.g., the following phase convention [2] — [5]

$$\Theta|\mathbf{1}> \stackrel{\text{def}}{=} -|\mathbf{2}>, \quad \Theta|\mathbf{2}> \stackrel{\text{def}}{=} -|\mathbf{1}>, \tag{28}$$

and taking into account that $<\psi|\varphi> = <\Theta\varphi|\Theta\psi>$, one easily finds from (22) – (27) that

$$h_{11}^{LOY} = h_{22}^{LOY} \tag{29}$$

in the CPT–invariant system. This is the standard result of the LOY approach and this is the picture which one meets in the literature [1] — [9], [18]. So, within this approximation the masses of particle "1" and its antiparticle "2" are equal in the system preserving CPT symmetry,

$$M_{11}^{LOY} = M_{22}^{LOY}. \tag{30}$$

2.2 Beyond the LOY approximation.

The approximate formulae for $H_{\parallel}(t)$ (where $t \geq 0$) have been derived in [16, 17] assuming that

$$[P, H] \neq 0, \tag{31}$$

and using the Królikowski–Rzewuski (KR) equation for the projection of a state vector [11, 12]. Such an approach is convenient when one searches for the properties of the systems prepared at some initial instant t_0, say, $t_0 = 0$, and then evolving in time $t \geq t_0 = 0$ [12]. In such cases the S–matrix formalism mentioned at the end of Sec.1 does not work (it works when time t varies from $t = -\infty$ to $t = +\infty$), and all conclusions following from the scattering theory need not be true for these systems.

The KR Equation results from the Schrödinger equation (10) for the total system under consideration, and, in the case of initial conditions of the type (11), takes the following form

$$(i\frac{\partial}{\partial t} - PHP)U_{\parallel}(t) = -i\int_0^{\infty} K(t-\tau)U_{\parallel}(\tau)d\tau, \tag{32}$$

where $U_{\parallel}(0) = P$ and $U_{\parallel}(t<0) = 0$ [17],

$$K(t) = \mathit{\Theta}(t) PHQ \exp(-itQHQ)QHP, \tag{33}$$

and $\Theta(t) = \{1 \text{ for } t \geq 0, \quad 0 \text{ for } t < 0\}$. On the other hand, in this case simply

$$U_{\|}(t) \equiv PU(t)P = Pe^{-itH}P, \quad (t \geq t_0). \tag{34}$$

Defining

$$H_{\|}(t) \stackrel{\text{def}}{=} PHP + V_{\|}(t), \tag{35}$$

one finds from (3), (5) and (32)

$$V_{\|}(t)U_{\|}(t) = -i \int_0^{\infty} K(t-\tau)U_{\|}(\tau)d\tau \stackrel{\text{def}}{=} -iK * U_{\|}(t). \tag{36}$$

(Here the asterisk $*$ denotes the convolution: $f * g(t) = \int_0^{\infty} f(t-\tau)g(\tau)\, d\tau$). Next, using this relation and a retarded Green's operator $G(t)$ for the equation (32)

$$G(t) = -i\Theta(t) \exp(-itPHP)P, \tag{37}$$

one obtains [12, 16, 17]

$$V_{\|}(t)\ U_{\|}(t) = -iK * \Big[\mathit{1} + \sum_{n=1}^{\infty} (-i)^n L * \ldots * L\Big] * U_{\|}^{(0)}(t), \tag{38}$$

where L is convoluted n times, $\mathit{1} \equiv \mathit{1}(t) \equiv \delta(t)$,

$$L(t) = G * K(t), \tag{39}$$

and

$$U_{\|}^{(0)} = \exp(-itPHP)\ P \tag{40}$$

is a "free" solution of Eq. (32). Of course, the series (38) is convergent if $\| L(t) \| < 1$. If for every $t \geq 0$

$$\| L(t) \| \ll 1, \tag{41}$$

then, to the lowest order of $L(t)$, one finds from (38) [12, 16, 17]

$$V_{\|}(t) \cong V_{\|}^{(1)}(t) \stackrel{\text{def}}{=} -i \int_0^{\infty} K(t-\tau) \exp\left[i(t-\tau)PHP\right] d\tau, \quad (t \geq 0). \tag{42}$$

From (36) and (42) it follows that

$$V_{\|}(t=0) \equiv V_{\|}^{(1)}(t=0) = 0, \tag{43}$$

which (by (35)) means that

$$H_{\parallel}(t=0) \equiv PHP, \tag{44}$$

and thus in the general case

$$h_{jk}(t=0) \equiv H_{jk}, \quad v_{jk}(t=0) \equiv 0, \tag{45}$$

where $v_{jk}(t) =< \mathbf{j}|V_{\parallel}(t)|\mathbf{k} >$, $j,k=1,2$.

In the case of (14) of the projector P, the approximate formula (42) for $V_{\parallel}(t)$ enables us to calculate the matrix elements $v_{jk}(t>0)$ of $V_{\parallel}(t) \cong V_{\parallel}^{(1)}(t)$ [16, 17], which leads to the following expressions for $v_{jk}(t \to \infty) \stackrel{\text{def}}{=} v_{jk}$ (for details see [16, 17]),

$$\begin{aligned}
v_{j1} = & - \frac{1}{2}\Big(1+\frac{H_z}{\kappa}\Big)\Sigma_{j1}(H_0+\kappa) - \frac{1}{2}\Big(1-\frac{H_z}{\kappa}\Big)\Sigma_{j1}(H_0-\kappa) \\
& - \frac{H_{21}}{2\kappa}\Sigma_{j2}(H_0+\kappa) + \frac{H_{21}}{2\kappa}\Sigma_{j2}(H_0-\kappa), \\
v_{j2} = & - \frac{1}{2}\Big(1-\frac{H_z}{\kappa}\Big)\Sigma_{j2}(H_0+\kappa) - \frac{1}{2}\Big(1+\frac{H_z}{\kappa}\Big)\Sigma_{j2}(H_0-\kappa) \\
& - \frac{H_{12}}{2\kappa}\Sigma_{j1}(H_0+\kappa) + \frac{H_{12}}{2\kappa}\Sigma_{j1}(H_0-\kappa),
\end{aligned} \tag{46}$$

where $j,k=1,2$,

$$H_z = \frac{1}{2}(H_{11}-H_{22}), \tag{47}$$

and

$$H_0 = \frac{1}{2}(H_{11}+H_{22}),, \tag{48}$$

$$\kappa = (|H_{12}|^2 + H_z^2)^{1/2}. \tag{49}$$

Hence, by (35)

$$h_{jk} = H_{jk} + v_{jk}, \tag{50}$$

which defines the operator $H_{\parallel} \stackrel{\text{def}}{=} H_{\parallel}(t \to \infty) \stackrel{\text{def}}{=} PHP + V_{\parallel}^{(1)}(\to \infty) \equiv H_{\parallel}(\infty)$. It should be emphasized that all the components of the expressions (46) are of the same order with respect to $\Sigma(\epsilon)$.

These formulae for v_{jk} and thus for h_{jk} have been derived without assuming any symmetries of the type CP–, T–, or CPT–symmetry for the total Hamiltonian H of the system considered. According to the general ideas of the quantum theory, one can state that the operator $H_{\|}(t \rightarrow \infty) \equiv H_{\|}(\infty)$ describes the bounded or quasistationary states of the subsystem considered.

The same formula for $H_{\|}(\infty)$ can be obtained by improving the method described in [1, 2]. Namely, analysing the LOY derivation of the effective Hamiltonian for the neutral kaon complex one can observe that the components containing the matrix elements $< \mathbf{j}|H_I|\mathbf{k} >$, $(j,k = 1,2)$ are neglected in the right sides of Eqs (18), (19) in [1]. It is found in [35] that such an approximation is not true for the whole domain of the parameter t: It is not true for $t \simeq t_0 = 0$. The formulae for the improved effective Hamiltonian H_{LOY}^{Imp} can be obtained by considering the equations for the component $|\psi >_{\|}$ of the state vector $|\psi >$ instead of the LOY equations for the amplitudes a_1, a_2 (see Eq (6)) mentioned above. This approach takes into account the matrix elementsof H_I which were neglected in the LOY paper [1]. It appears that such H_{LOY}^{Imp} equals $H_{\|}(\infty)$ exactly [35].

Properties of the matrix elements h_{jk} of the apprroximate $H_{\|}(t)$ described in this Subsection have been examined in [16] for the generalized Fridrichs–Lee model [18]. For this model it has been found in [16] that $h_{jk}(t) \simeq h_{jk}$ practically for $t \geq T_{as} \simeq \frac{10^2}{\pi(m_0 - |m_{12}| - \mu)}$, where $m_0 \equiv H_{11} = H_{22}$, $m_{12} \equiv H_{12}$ and $(m_0 - \mu)$ is the the diffrence between the mass of the unstable particles considered and the threshold energy of the continuum state of decay products (see [16], formula (153)). For the neutral K–system, to estimate T_{as} it has been taken $(m_0 - |m_{12}| - \mu) \simeq (m_0 - \mu) = m_K - 2m_\pi \sim 200$MeV, which gives $T_{as} \sim 10^{-22}$ sec [19].

3 Consequences of CPT–invariance.

In the case of preserved CPT symmetry,

$$\Theta H \Theta^{-1} = H, \tag{51}$$

and violated CP,

$$[\mathcal{CP}, H] \neq 0, \tag{52}$$

assuming (28) one finds

$$H_{11} = H_{22}, \tag{53}$$

which implies that for $t = 0$ (see (45))

$$h_{11}(0) \equiv h_{22}(0). \tag{54}$$

This property means that

$$\Re\left(h_{11}(0)\right) = M_{11}(0) \equiv \Re\left(h_{22}(0)\right) = M_{22}(0), \tag{55}$$

i.e., in CPT–invariant system a particle and its antiparticle are created at $t = 0$ as quantum objects with equal masses.

From (53) it also follows that $\kappa \equiv |H_{12}|$, $H_z \equiv 0$ and $H_0 \equiv H_{11} \equiv H_{22}$, and [16, 17]

$$\Sigma_{11}(\varepsilon = \varepsilon^*) \equiv \Sigma_{22}(\varepsilon = \varepsilon^*) \stackrel{\text{def}}{=} \Sigma_0(\varepsilon = \varepsilon^*). \tag{56}$$

Therefore for $t \to \infty$ the matrix elements v_{jk}^{Θ} (46) of operator $V_{\parallel}^{\Theta}$ ($V_{\parallel}^{\Theta}$ denotes $V_{\parallel}$ when (51) occurs) take the following form

$$\begin{aligned} v_{j1}^{\Theta} = & - \frac{1}{2}\Big\{\Sigma_{j1}(H_0 + |H_{12}|) + \Sigma_{j1}(H_0 - |H_{12}|) \\ & + \frac{H_{21}}{|H_{12}|}\Sigma_{j2}(H_0 + |H_{12}|) - \frac{H_{21}}{|H_{12}|}\Sigma_{j2}(H_0 - |H_{12}|)\Big\}, \\ v_{j2}^{\Theta} = & - \frac{1}{2}\Big\{\Sigma_{j2}(H_0 + |H_{12}|) + \Sigma_{j2}(H_0 - |H_{12}|) \\ & + \frac{H_{12}}{|H_{12}|}\Sigma_{j1}(H_0 + |H_{12}|) - \frac{H_{12}}{|H_{12}|}\Sigma_{j1}(H_0 - |H_{12}|)\Big\}. \end{aligned} \tag{57}$$

Using these relations one easily finds

$$\begin{aligned} h_{11}^{\Theta} - h_{22}^{\Theta} = & - \frac{1}{2|H_{12}|}\Big[H_{21}\,\Sigma_{12}(H_0 + |H_{12}|) - H_{12}\,\Sigma_{21}(H_0 + |H_{12}|) \\ & - H_{21}\,\Sigma_{12}(H_0 - |H_{12}|) + H_{12}\,\Sigma_{21}(H_0 - |H_{12}|)\Big], \end{aligned} \tag{58}$$

where h_{jk}^{Θ} is defined analogously to v_{jk}^{Θ}. From this Equation it follows that within the more accurate approximation than the one used by Lee, Oehme and Yang there is

$$h_{11}^{\Theta} - h_{22}^{\Theta} \neq 0. \tag{59}$$

So, the more exact approximation does not confirm the standard LOY result (29). Expression (58) also impairs a conviction that a particle and its antiparticle have always the same mass in CPT–invariant system, i.e., the conclusion

(30) of the LOY theory. Namely, taking into account the decomposition (25) it is not difficult to obtain the following relation from (58)

$$\begin{aligned} \Re\,(h_{11}^{\Theta} - h_{22}^{\Theta}) &= \Im\Big\{\frac{H_{21}}{|H_{12}|}\,\Sigma_{12}^{I}(H_0 + |H_{12}|)\Big\} \\ &- \Im\Big\{\frac{H_{21}}{|H_{12}|}\,\Sigma_{12}^{I}(H_0 - |H_{12}|)\Big\} \\ &\stackrel{\text{def}}{=} M_{11} - M_{22} \equiv \Delta m, \end{aligned} \tag{60}$$

which means that the masses of a particle and its simultaneously prepared antiparticle need not be equal at $t \rightarrow \infty$ if CPT–symmetry holds. Only at $t = 0$, i.e., at the moment of the creation of a particle and its antiparticle their masses can be equal (55). The sign of Δm depends on the form of H.

It can be expected that similar conclusions can be drawn using the approach exploited in [7], where perturbation theory correction improving H_{LOY} have been considered.

4 Possible experimental consequences.

It seems that the only way for an experimantal verification of the effect expressed by formula (60) in the near future is to search for the properties of $K_0, \overline{K_0}$ and similar complexes. The real part of $(h_{11} - h_{22})$ can be expressed by the parameters measured in the experiments with neutral K, or B mesons [3] — [6], [36], that is, it is an experimentally measurable quantity. Within the use of the formalism described in Sec. 2.2, $(h_{11} - h_{22})$ has been estimated for the generalized Fridrichs–Lee model [18]. Assuming CPT–invariance (i.e., (51)) and $|m_{12}| \equiv |H_{12}| \ll (m_0 - \mu) \equiv (H_0 - \mu)$ it has been found in [19] that

$$\begin{aligned} \Re\,(h_{11}^{FL} - h_{22}^{FL}) &\simeq i\frac{m_{21}\Gamma_{12} - m_{12}\Gamma_{21}}{4(m_0 - \mu)} \\ &\equiv \frac{\Im\,(m_{12}\Gamma_{21})}{2(m_0 - \mu)}, \end{aligned} \tag{61}$$

where h_{jj}^{FL}, $(j = 1, 2)$, denotes h_{jj} calculated for the Fridrichs–Lee model. For the $K_0, \overline{K_0}$–complex $\Gamma_{21} \equiv \Gamma_{12}^{*} \simeq \Re\,(\Gamma_{12}) \simeq \frac{1}{2}\Gamma_s \sim 3,7 \times 10^{-12}$MeV.

This property and relation (61) enable us to find the following estimation of $\Re\,(h_{11}^{FL} - h_{22}^{FL})$ for the neutral K–system [19]

$$\Re\,(h_{11}^{FL} - h_{22}^{FL}) \sim 9,25 \times 10^{-15} \Im\,(m_{12}) \equiv 9,25 \times 10^{-15} \Im\,(H_{12}). \qquad (62)$$

The assumptions leading to (61) can be used to obtain

$$h_{11}^{FL} + h_{22}^{FL} \simeq 2m_0 - i\Gamma_0 - \frac{i}{2}\frac{\Re\,(H_{21}\Gamma_{12})}{m_0 - \mu}, \qquad (63)$$

(where $\Gamma_0 \equiv \Gamma_{11} = \Gamma_{22}$) which and (61), (62) give

$$\frac{\Re\,(h_{11}^{FL} - h_{22}^{FL})}{\Re\,(h_{11}^{FL} + h_{22}^{FL})} \simeq \frac{\Im\,(H_{12})}{8m_0(m_0 - \mu)}\Gamma_s \sim 9,25 \times 10^{-18}\,(\Im\,(H_{12}))\;[\mathrm{MeV}]^{-1}, \quad (64)$$

(where $m_0 = m_K$ is inserted). This estimation does not contradict the experimental data for neutral K mesons [36].

From (62) it follows that the effect described by relation (60) is very, very small indeed, and it is, probably, beyond the accuracy of today's experiments [36]. Test of more higher accuarcy are expected to be performed in the near future [6]. Advances in the experimental methods are planned in DAΦNE project at Frascati (Rome) and BARBAR at SLAC (Stanford) to measure the smallobservables for neutral meson systems. So, there is a chance that the effects described in Sec. 3 can be confirmed experimentally in the near future.

Using the formalism described in Sec. 2.2 it has been shown in [16, 17, 37] that eigenvectors $|l^t>, |s^t>$,

$$\begin{aligned} |l^t> \;&= \tfrac{1}{(1+|\alpha_l(t)|^2)^{1/2}}[|\mathbf{1}> - \alpha_l(t)|\mathbf{2}>], \\ |s^t> \;&= \tfrac{1}{(1+|\alpha_s(t)|^2)^{1/2}}[|\mathbf{1}> - \alpha_s(t)|\mathbf{2}>], \end{aligned} \qquad (65)$$

(for the definitions of $\alpha_l(t), \alpha_s(t)$ see [16]) of the more accurate effective Hamiltonian, $H_{\|}(t)$, for the neutral K mesons complex possess the property $|l(s)^{t=0}> \neq |l(s)^{t\to\infty}> \stackrel{\mathrm{def}}{=} |l(s)>$ if the CP symmetry is violated (i.e., if (52) holds). This property is the consequence of the fact that $H_{\|}(t=0) \neq H_{\|}(t \to \infty)$. For the eigenstates $|K_{L(S)}>$ of H_{LOY} such an effect is absent. (The eigenstates $|l^t>, |s^t>$ for $H_{\|}(t)$ correspond to the eigenvectors $|K_{L(S)}>$ for H_{LOY}). The property of $|l^t>, |s^t>$ mentioned above means

that the real properties of the system created at $t_0 = 0$ should be different at $t = t_0 = 0$ and at $t \gg t_0$, i.e., at $t \to \infty$. Relations (54), (55) and (59), (60) are a consequnence of this effect. Another implication of the above is that $| < l(s)^{t=0}|l(s) > | \neq 1$, although $< l(s)^{t=0}|l(s)^{t=0} >=< l(s)|l(s) >= 1$. The possibility to verify of this effect by an experiment has been discussed in [16, 17, 37]. The following problem arises: The states $|l^{t>0} >, |s^{t>0} >$ are not orthogonal, $< l^{t>0}|s^{t>0} >\neq 0$, and similarly $< t; l^t|s^t; t > |_{t>T_{as}} \equiv < t; l|s; t > |_{t>T_{as}} \neq 0$, where $|l(s); t > |_{t>T_{as}} \simeq \exp(-itH_{\|})|l(s) > |_{t>T_{as}}$. This means that at time t an observer is unable to register separatelly decay products of the state $|s^t >$ and of $|l^t >$ [17]. Nevertheless, it seems that this difficulty can be overcome. Namely, taking into account that for the neutral K mesons system $\tau_{l(s)} \gg T_{as}$ and $\tau_l \simeq 0,58 \times 10^2 \tau_s$, [36], one finds $| < t; l|s; t > |^2_{t \sim \tau_s} \sim e^{-1} \; | < l|s > |^2 \simeq e^{-1} \; | < K_L|K_S > |^2$ and $| < t; l|s; t > |^2_{t \sim \tau_l} \sim e^{-1} \, (e^{-0.58})^{100} \, | < l|s > |^2 \simeq e^{-1}(e^{-0.58})^{100} \, | < K_L|K_S > |^2 \cong 0$. This last estimation menas that for times $t \sim \tau_l$ and $t > \tau_l$ the result of the observation of the decay process of states $|l >$ is practically not disturbed by the decay of states $|s >$. Now we can find the probability $p_l(t)$ of finding the system in the particle state of type $|l^t >$ at a relatively long time, $t \gg T_{as}$, if it were in the state of this same type at $t = 0$, (i.e., in $|l^{t=0} >$). It equals [16, 37]

$$\begin{aligned} p_l(t \gg T_{as}) &= \simeq | < l^{t=0}|e^{-itH_{\|}}|l > |^2|_{t \gg T_{as}} \\ &= \exp(-t\Gamma_l)| < l^{t=0}|l > |^2. \end{aligned} \tag{66}$$

From this, the conclusion that there is a chance of observing a new effect which is absent in the standard LOY theory was drawn in [37]. Simply, within the LOY approach one has

$$\begin{aligned} p_l(t) \equiv p_{K_L}(t) &= | < K_L^{t=0}|e^{\left(-itH_{LOY}\right)}|K_L > |^2 \\ &\equiv | < K_L|e^{\left(-itH_{LOY}\right)}|K_L > |^2 \simeq \exp(-t\Gamma_l). \end{aligned} \tag{67}$$

Now, if one has the possibility of determing the function $p_l(t)$ directly from the experiment, then starting from this experimentally determined $p_l(t)$ one can draw a segment of a straight line for $t \sim \tau_l$ and $t > \tau_l$,

$$y_l(t) \stackrel{\text{def}}{=} \ln p_l(t) + b_l, \quad (t \sim \tau_l, t > \tau_l), \tag{68}$$

where

$$b_l = \ln | < l^{t=0}|l > |^2. \tag{69}$$

(The time $t = 0$ is the instant of the creation of the neutral K meson). Then one can produce this segment extrapolating it to the intersection with the y–axis. The point $y(0)$ determines the value of the parameter b_l. The approximation described in Sec. 2.2 (which is more accurate than the LOY method) predicts that there should be $b_l \neq 0$ in the CPT invariant but CP noninvariant system, whereas within the LOY theory one finds $b_l^{LOY} = 0$. This situation is presented in Fig. 1. In this case $\tan \varphi = \Gamma_l$.

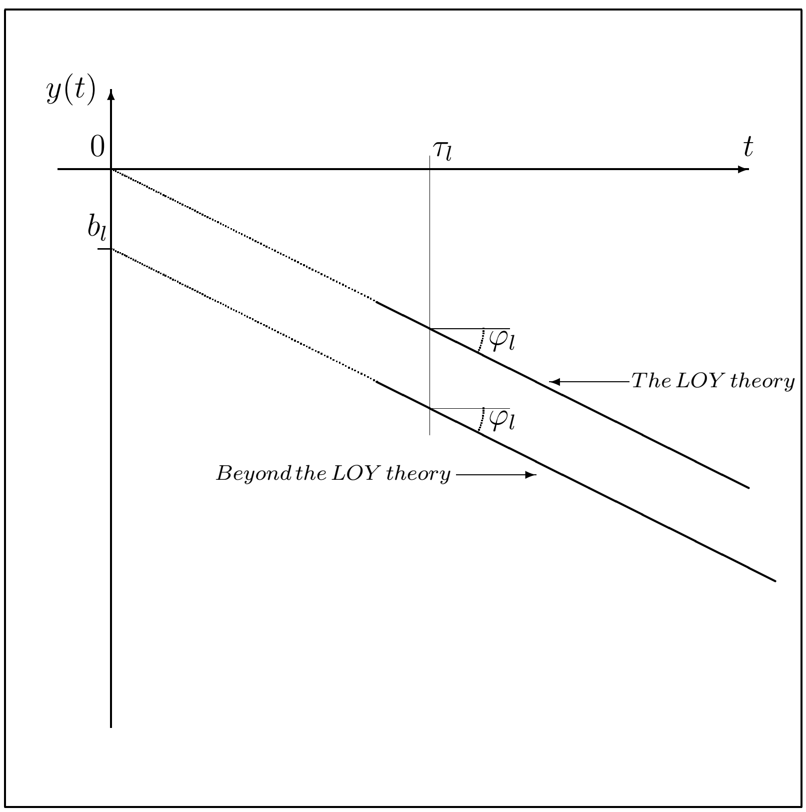

Figure 1: The hypothetical form of the extrapolated experimentally obtained (bold line) curve $y_l(t) = \ln p_l(t) = -\Gamma_l t + b_l$.

The following estimation has been found for the value of the parameter b_l in [37]: $b_l \simeq -2.67 \times 10^{-6}$.

It can be easily shown that if the property (51) holds then $H_{\|} = H_{LOY}$ if and only if $|H_{12}| = 0$, and similarly, following [19], that $(h_{11} - h_{22}) = 0$ if and only if $|H_{12}| = 0$. This means that if $|H_{12}| \equiv |< \mathbf{1}|H_I|\mathbf{2} > | \neq 0$ then there should be $H_{\|} \neq H_{LOY}$ and thus $b_l \neq 0$.

Any experimental confirmation of this effect, i.e, that $b_l \neq 0$, will mean, contrary to the predictions of the LOY theory, that if CP symmetry is violated then the properties of the system at the instant of its creation, $t = t_0 \equiv 0$, and at time $t \gg t_0$ ($t \to \infty$) are different. Indirectly, after verifying if $H_{12} \neq 0$, it will also mean that the other conclusions derived within the use the more accurate approximation described in Sec. 2.2, (including results obtained in Sec. 3) should be true.

There is another relation which seems to be useful when one tries to verify predictions based on the aproximation described in Sec. 2.2. Namely, one can find that

$$\alpha_l + \alpha_s = \frac{h_{11} - h_{22}}{h_{12}}, \tag{70}$$

(see [16, 17]). Within the LOY theory one simply has $\alpha_l^{LOY} \stackrel{\text{def}}{=} \alpha^{LOY} = -\alpha_s^{LOY}$, that is, $\alpha_l^{LOY} + \alpha_s^{LOY} = 0$. So, if $(h_{11} - h_{22}) \neq 0$ then measurements determining the values of the parameters α_l and α_s more accurately than it is possible in todays experiments should confirm this property.

5 Discussion.

5.1 General remarks.

The real parts of the diagonal matrix elements of the mass matrix $H_{\|}$, h_{11} and h_{22}, are considered in the literature as masses of particles $|\mathbf{1}>, |\mathbf{2}>$ (eg., mesons K_0 and $\overline{\mathrm{K}}_0$, etc.,) [1] — [9]. Result (60) means that if $\Sigma^I_{jk}(H_0 + |H_{12}|) \neq 0$, or if $\Sigma^I_{jk}(H_0 - |H_{12}|) \neq 0$, $(j, k = 1, 2)$, or if both these cases occur, i.e., if particle "1" and antiparticle "2" are unstable and if they were simultneously prepared at $t = t_0 \equiv 0$, then at $t \gg t_0$ (in particular at $t \to \infty$) the masses of a decaying particle "1" and its antiparticle "2" should be different if CPT–symmetry is conserved and CP is violated in the system containig these unstable particles. In other words, unstable states $|\mathbf{1}>, |\mathbf{2}>$

appear to be nondegenerate in mass if CPT–symmetry holds and CP does not in the total system considered.

On the other hand, examining properties of the operator $\Sigma_{jk}^{I}(\epsilon)$ (27) one finds that

$$\Sigma_{jk}^{I}(H_0 \pm |H_{12}|) = 0 \text{ if } (H_0 \pm |H_{12}|) < \varepsilon_M, \tag{71}$$

where ε_M denotes the lower bound for the continous part $\sigma_c(QHQ)$ of the spectrum $\sigma(QHQ)$ of the operator QHQ. States $|\mathbf{1}>, |\mathbf{2}>$, for which this property occur, correspond to bound states and they cannot decay at all. This observation and relation (60) mean that in the CPT–invariant system the masses of a given particle and its aniparticle prepared at the finite instant $t = t_0 > -\infty$ are equal (i.e., apear to be degenerate) only in the case of bound (stable) states $|\mathbf{1}>, |\mathbf{2}>$. The case when vectors $|\mathbf{1}>, |\mathbf{2}>$ describe pairs of particles $p, \overline{p}$, or e^-, e^+, can be considered as an example of such states.

In fact there is nothing strange in these conlusions. From (51) (or from the CPT Theorem [31]) it only follows that the masses of particle and antiparticle eigenstates for H (i.e., masses of stationary states of H) should be the same in CPT invariant system. Such a conclusion can not be derived from (51) for particle $|\mathbf{1}>$ and its antiparticle $|\mathbf{2}>$ if they are unstable, i.e., if states $|\mathbf{1}>, |\mathbf{2}>$ are not eigenstates of H. One should remember that the CPT Theorem of axiomatic quantum field theory has been proved for quantum fields corresponding to stable quantum objects. Only such fields are considered in axiomatic quantum field theory [31]. There is no axiomatic quantum field theory of unstable quantum particles. So, all implications of CPT Theorem (including those obtained within the S–matrix method) need not be valid for decaying particles prepared at some initial instant $t_0 = 0$ and then evolving in time $t \geq 0$. Simply, the consequences of CPT invariance need not be the same for systems in which time t varies from $t = -\infty$ to $t = +\infty$ and for the system in which t can vary only from $t = t_0 > -\infty$ to $t = +\infty$.

The following conclusion can be drawn from (55) and (60): The time evolution causes the masses of the unstable particle and its simultaneously prepared at $t = 0$ antiparicle to be different at $t > T_{as}$ in the CPT invariant but CP noninvariant system.

All the above conclusions contradict the standard result of the LOY and related approaches. Properties of the real systems described by formulae (58), (60) are unobservable for the LOY approximation. On the other hand,

these relations are not in conflict with the general conclusions of [13, 14], where CPT–transformation properties of the exact $H_{\|}(t)$ have been studied. Confronting relation (29) with (58) one should remember that, in fact, the LOY model is unable to describe real properties of the system considered [19, 20, 22]. Namely, it has been proved in [19, 20] that systems containing an exponentially decaying subsystem (i.e., evolving in time according to (21)) cannot be CPT–invariant. A similar conclusion can be drawn from the results obtained in [22]. This means that the CPT–symmetry properties of the LOY model need not reflect real properties of the system considered. At the same time from [13, 14] it follows that CPT– and other transformation properties of the effective Hamiltonian described in Sec. 2.2 are consistent with properties of the real systems.

Note that considering in detail the generalized Fridrichs–Lee model one finds that $\Gamma_{jk} = 0$, $(j,k = 1,2)$ for $m_0 < \mu$, i.e., for bound states [18, 16]. This observation and relation (61) imply that for bound (stable) states $\Re\,(h_{11}^{FL} - h_{22}^{FL}) = 0$. So, if CPT–symetry is conserved in this model, then particle and antiparticle bound states remain to be also degenerate in mass beyond the LOY approximation, whereas unstable states (i.e., states for which $m_0 > \mu$) appear to be nondegenerate in mass in this model if CPT–symetry holds but CP does not. These observations confirm our earlier conlusions following from (60), (71).

As it was mentioned in Sec. 4, one can conclude from (62), (64) that the relation (60) leads to a very small value of Δm, which is probably, for single "particle–aniparticle" pair, beyond today's experiments accuracy. Nevertheless, this effect should be significant if the number of pairs of unstable particles and their antiparticles is very large. Such conditions can be met at the origin immediately after the "Big Bang", at the first instants of the existence of our Universe [23] — [30]. (Note, that the initial condition (4) corresponds exactly to the case of the creation of the Universe).

5.2 Possible cosmological applications.

In [26] — [30] an observation has been used that a baryon asymmetry could arise even in thermal equilibrium if CPT symmetry is violated. It is assumed in such theories that CPT is not realized as a good symmetry in the early Universe. From the main result of Sec. 3, (60), it follows that in fact CPT need not be violated in order that the baryon asymmetry could occur. Indeed,

the termodynamic observable number density, $n_X^{eq,\pm}$, in thermal equilibrium, is a function of temperature alone:

$$n_X^{eq,\pm}(T) = g_X^s \int \frac{d^3\vec{p}}{(2\pi)^3} f_X^{\pm}(\vec{p},T), \tag{72}$$

where g_X^s is the number of spin states of particle type X,

$$f_X^{\pm}(\vec{p},T) = \frac{1}{e^{(E_X-\mu_X)/T} \pm 1}, \tag{73}$$

is the equilibrium phase space occupancy, μ_X is a possible chemical potential of the particles considered, $E_X = \sqrt{m^2+p^2}$ and $\vec{p}$ is the momentum, $p = |\vec{p}|$, $\pm$ refers to Bose–Eistein $(-)$, and Fermi–Dirac $(+)$ statistics [23] — [30].

In the literature the relation

$$n_X^{eq,+}(T) = n_{\overline{X}}^{eq,+}(T), \tag{74}$$

which can be obtained from (73) assuming that masses, m, of particle, X, and $\overline{m}$ of antiparticle, $\overline{X}$, are equal, (i.e., that $E_X = E_{\overline{X}}$), and that baryon number B is not conserved, (which, in thermal equilibrium, imlpies $\mu_X = 0$ [23] — [30]), is considered as the suggestion that without some extra mechanisms (e.g., a violation of CPT) the particle–antiparticle asymmetry can not be produced.

Taking into account the results of Sec. 3 one finds that in fact there is $m \equiv m_t$, and

$$m_{t=0} = \overline{m}_{t=0}, \tag{75}$$

but

$$m_{t>T_{as}} \stackrel{\text{def}}{=} m \neq \overline{m}_{t>T_{as}} \stackrel{\text{def}}{=} \overline{m}, \tag{76}$$

and thus $E_X \equiv \sqrt{m^2+p^2} \neq E_{\overline{X}} \equiv \sqrt{\overline{m}^2+p^2}$ in the case of unstable particles X, which implies $n_X^{eq,\pm}(T) \neq n_{\overline{X}}^{eq,\pm}(T)$ instead of (74). The relation $n_X^{eq,\pm}(T) \neq n_{\overline{X}}^{eq,\pm}(T)$ means that the asymmetric number of particles X, and antiparticles $\overline{X}$, can be generated at time $t > T_{as} \gg t = t_0 = 0$ in a CPT invariant but CP noninvariant system even when there is no the extra mechanism in this system and even were the symetric numbers, $\mathcal{N}_t^X$ of X, and $\mathcal{N}_t^{\overline{X}}$ of $\overline{X}$ at time $t = t_0 = 0$:

$$\mathcal{N}_{t=0}^X = \mathcal{N}_{t=0}^{\overline{X}}. \tag{77}$$

If to take into account that in the observed Universe $\mathcal{N}_t^X \gg \mathcal{N}_t^{\overline{X}}$, the following conclusion seems to be obvious

$$m_{t>0} \ < \ \overline{m}_{t>0}. \tag{78}$$

Now let us examine the supposition that asymmetric numbers of particles and antiparticles can be generated at time $t > T_{as}$ in thermal equilibrium. Only the simplest nontrivial example will be considered in this paper. From the results of Sec. 3 it follows that stable particles cannot produce any contribution into the diference of masses particles and antiparticles but only unstable particles can generate the matter–antimatter masses asymmetry. So, we will consider only unstable X. Assuming that the baryon number B is not conserved we will take $\mu_X = 0$ [23] — [30], (In general, for most purposes it is reasonable to set μ to be zero [24]). Thus the number density, $n_X^{eq,\pm}$, of unstable particles X in thermal equilibrium state equals

$$\begin{aligned} n_X^{eq,\pm}(T) &= g_X^s \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{e^{\frac{E_X}{T}} \pm 1} \\ &= \frac{2g_X^s}{(2\pi)^2} \int_m^\infty \frac{E_X\sqrt{E_X^2 - m^2}}{e^{\frac{E_X}{T}} \pm 1} dE_X \\ &\equiv \frac{g_X^s}{2\pi^2} T^3 I_N^{\pm}(a), \end{aligned} \tag{79}$$

where $a = \frac{m}{T} \ > \ 0$, and

$$I_N^{\pm}(a) \stackrel{\text{def}}{=} \int_a^\infty \frac{x\sqrt{x^2 - a^2}}{e^x \pm 1} dx. \tag{80}$$

Similarly, for the number density $n_{\overline{X}}^{eq,\pm}$ of antiparicles $\overline{X}$ one finds

$$n_{\overline{X}}^{eq,\pm}(T) = \frac{g_X^s}{2\pi^2} T^3 I_N^{\pm}(\overline{a}), \tag{81}$$

where $\overline{a} = \frac{\overline{m}}{T}$.

From the observations of our Universe it follows that $n_X^{eq,\pm}(T) \ > \ n_{\overline{X}}^{eq,\pm}(T)$ at time $t \gg T_{as}$. It can occur in thermal equilibrium only if $m < \overline{m}$. So tere should be $\overline{m} = m + \Delta m$, and $\Delta m > 0$. Now if we take into account

the estimation (64), from which it follows that Δm should be much, much smaller than m and $\overline{m}$, and define

$$\sigma \stackrel{\text{def}}{=} \frac{\Delta m}{T},$$

then we find for $\Delta m \ll m$, i.e., for $a \equiv \frac{m}{T}$, $\overline{a} = \frac{\overline{m}}{T}$ and $\sigma \ll a$

$$I_N^{\pm}(\overline{a}) \equiv I_N^{\pm}(a+\sigma) \simeq I_N^{\pm}(a) + \sigma I_{\Delta N}^{\pm}(a), \tag{82}$$

where

$$I_{\Delta N}^{\pm}(z) \stackrel{\text{def}}{=} \frac{\partial I_N^{\pm}(x)}{\partial x}\Big|_{x=z}. \tag{83}$$

This means that the difference between the number densities of particles and antiparticles in the thermal equilibrium should be approximately equal

$$n_X^{eq,\pm}(T) - n_{\overline{X}}^{eq,\pm}(T) \simeq -\frac{g_X^s}{2\pi^2} T^3 \Big(\frac{\Delta m}{T}\Big) I_{\Delta N}^{\pm}(a)\big|_{a=\frac{m}{T}}. \tag{84}$$

Using this relation the following ratio can be found

$$\begin{aligned} \eta_x^{\pm} \stackrel{\text{def}}{=} \frac{n_X^{eq,\pm}(T) - n_{\overline{X}}^{eq,\pm}(T)}{n_X^{eq,\pm}(T)} &= -\frac{\Delta m}{T} \frac{I_{\Delta N}^{\pm}(a)}{I_N^{\pm}(a)}\Big|_{a=\frac{m}{T}} \\ &\equiv -\frac{\Delta m}{T} \Big\{\frac{\partial}{\partial z} \ln I_N^{\pm}(z)\Big\}\Big|_{z=\frac{m}{T}}. \end{aligned} \tag{85}$$

Taking into accout the result (A.11) the difference (84) between number densities takes the form

$$n_X^{eq,\pm}(T) - n_{\overline{X}}^{eq,\pm}(T) \simeq \mp \frac{g_X^s}{2\pi^2} m^2\, \Delta m \sum_{n=1}^{\infty} (\mp 1)^n K_1(an)\big|_{a=\frac{m}{T}}. \tag{86}$$

At the same time, the results (A.11) and (A.10) lead to the following expression for the ratio $\eta_x^{\pm}$

$$\eta_x^{\pm} = \frac{\Delta m}{T} \frac{\sum_{n=1}^{\infty} (\mp 1)^n K_1(an)}{\sum_{n=1}^{\infty} \frac{(\mp 1)^n}{n} K_2(an)}\Bigg|_{a=\frac{m}{T}}. \tag{87}$$

(The functions $K_1(x), K_2(x)$, which appear in the last two formuale, are the modified Bessel functions [43]).

Relations (86) and (87) are rather inconvinient. So discussing the problem whether the effect described at the end of Sec. 3 by formula (60) is able to generate a significant contribution to the observed baryon–antibaryon asymmetry or not, it is sufficient to consider the simplest lower and upper bounds for the ratio $\eta^{\pm}_{x}$.

The estimations given in Appendix B lead to the conclusion that the ratio $\eta^{\pm}_{x}$ (85) has the follwing simplest (and rather rough) lower and upper bounds

$$\eta^{\pm}_{x,min} \ < \ \eta^{\pm}_{x} \ < \ \eta^{\pm}_{x,mx}, \tag{88}$$

where

$$\eta^{\pm}_{x,mx} \ \stackrel{\rm def}{=} \ \frac{\Delta m}{T}\,\frac{[-I^{\pm}_{\Delta N,mx}(a)]}{I^{\pm}_{N,min}(a)}\Big|_{a=\frac{m}{T}}, \tag{89}$$

and

$$\eta^{\pm}_{x,min} \ \stackrel{\rm def}{=} \ \frac{\Delta m}{T}\,\frac{[-I^{\pm}_{\Delta N,min}(a)]}{I^{\pm}_{N,mx}(a)}\Big|_{a=\frac{m}{T}}, \tag{90}$$

So, in the case of fermions (baryons) the bounds (B.3) and (B.7) yield

$$\eta^{-}_{x,mx} \ = \ \frac{\Delta m}{T}\frac{1}{(1-e^{-a})^{2}}\frac{K_{1}(a)}{K_{2}(a)}\Big|_{a=\frac{m}{T}}, \tag{91}$$

which, within the use of (B.11), (B.12) gives for $\frac{m}{T} \ll 1$

$$\eta^{-}_{x,mx} \simeq \frac{\Delta m}{2m}, \tag{92}$$

and for $\frac{m}{T} \gg 1$,

$$\eta^{-}_{x,mx} \simeq \frac{\Delta m}{T}. \tag{93}$$

Analogously, taking into account (B.2) and (B.8) one finds

$$\eta^{-}_{x,min} \ = \ \frac{\Delta m}{T}(1-e^{-a})^{2}\frac{K_{1}(a)}{K_{2}(a)}\Big|_{a=\frac{m}{T}}. \tag{94}$$

From this expression and from (B.11), (B.12) it follows

$$\eta^{-}_{x,min} \ \simeq \ \frac{1}{2}\frac{\Delta m}{T}\Big(\frac{m}{T}\Big)^{3}, \quad (\tfrac{m}{T}\ll 1), \tag{95}$$

$$\eta^{-}_{x,min} \ \simeq \ \frac{\Delta m}{T}, \quad (\tfrac{m}{T}\gg 1). \tag{96}$$

Considering the case of bosons and using (B.5) and (B.9) one obtains

$$\eta^{+}_{x,mx} = \frac{\Delta m}{T}(1+e^{-a})^{2}\frac{K_1(a)}{K_2(a)}\bigg|_{a=\frac{m}{T}}. \qquad (97)$$

Thus, keeping in mind (B.11), (B.12), the following conclusions can be drawn

$$\eta^{+}_{x,mx} \simeq 2\frac{\Delta m}{T}\frac{m}{T}, \quad (\tfrac{m}{T}\ll 1), \qquad (98)$$

$$\eta^{+}_{x,mx} \simeq \frac{\Delta m}{T}, \quad (\tfrac{m}{T}\gg 1). \qquad (99)$$

The lower bound for η^{+}_{x} can be found using (B.4) and (B.10). One has

$$\eta^{+}_{x,min} = \frac{\Delta m}{T}\frac{1}{(1+e^{-a})^{2}}\frac{K_1(a)}{K_2(a)}\bigg|_{a=\frac{m}{T}}. \qquad (100)$$

By means of (B.11), (B.12) one can obtain the following estimations for $\eta^{+}_{x,min}$

$$\eta^{+}_{x,min} \simeq \frac{1}{8}\frac{\Delta m}{T}\frac{m}{T}, \quad (\tfrac{m}{T}\ll 1), \qquad (101)$$

$$\eta^{+}_{x,min} \simeq \frac{\Delta m}{T}, \quad (\tfrac{m}{T}\gg 1). \qquad (102)$$

Now let us consider, e.g., the case $\frac{m}{T} \gg 1$. One observes that in this case

$$\eta^{\pm}_{x,mx} = \eta^{\pm}_{x,min} \equiv \eta_x \simeq \frac{\Delta m}{T}. \qquad (103)$$

One can convert this formula into the following one

$$\frac{\Delta m}{m} = \eta_x\,\frac{T}{m}, \;\; (m \gg T), \qquad (104)$$

which is valid for unstable fermions (baryons or leptons) as well for bosons (mesons).

Theory of big bang nucleosynthesis, which very accurately predicts the abundances of all light elements, as well as the comparision of baryon and foton densities in the Universe, lead to the conlusion that the there should be the following bounds for the ratio η_x: $10^{-9} \geq \eta_x \geq 10^{-10}$ [23] — [29]. So, using the relations (103), or (104), and inserting there, eg., $\eta_x = \eta_{x,mx} = 10^{-9}$

one can obtain some allowed, numerical value for Δm. Thus one can verify if the effect described by the relation (60) can produce suitable Δm. From the formula (60) it follows that if H_0 and $|H_{12}|$ are suitable large and $|H_{12}| \sim |H_0|$, or even $|H_{12}| > |H_0|$ then the expression (60) for $\Delta m \equiv \Re\,(h_{11}^{\Theta} - h_{22}^{\Theta})$ can take the required values. Indeed, expanding $\Sigma_{12}(H_0 \pm |H_{12}|)$ by its Taylor series expansion one finds (to the lowest order of $|H_{12}|$)

$$\Delta m \equiv \Re\,(h_{11}^{\Theta} - h_{22}^{\Theta}) = 2\,\Im\,\Big(H_{21} \frac{\partial \Sigma_{12}^{I}(x)}{\partial x}\Big|_{x=H_0} \Big) \,+\, \ldots\,. \tag{105}$$

One should stress that due to the presence of resonace terms, the derivative $\frac{\partial \Sigma_{12}^{I}(x)}{\partial x}$ need not be small, and so the product $H_{21}\frac{\partial \Sigma_{12}^{I}(x)}{\partial x}$ in (105), especially if $|H_{12}|$ is suitable large. Note that the contribution of the higher terms of Taylor series expansion of $\Sigma_{12}(H_0 \pm |H_{12}|)$ into Δm can be even more large than (105) for suitably large $|H_{12}|$. The conditions guaranteeing the required magnitude of $|H_{12}|$ can be met at the first instants of the exsistence of our Universe. (The estimations (62), (64) refer to the today's epoch and to the case of neutral K mesons only). Note also that in the relation (104) the parameter η_x is multiplied by very smal factor $\frac{T}{m}$. Therefore it seems that the effect described by formulae (60) and (105) can have a significant contribution to the observed baryon — antibaryon asymmetry, and thus it is able, maybe together with one from the other mechanisms considered in [23] — [28], to explain the matter — antimatter asymmetry. A more detailed analysis of the cosmological consequences of the property (60) can be given in the future papers.

Note that the magnitude of $|H_{12}|$ and $|H_0|$ has not any effect on the validity of the approximate formulae (46) for matrix elements v_{jk} of $V_{\|} \simeq V_{\|}^{(1)}$, by means of which matrix elements h_{jk} of $H_{\|}$ are defined. The condition (39), 3which secures the validity of this approximation, does not depend on $|H_{12}|$. So, the effect expressed by the relation (60) occurs for large $|H_{12}|$ too.

6 Final remarks.

The approximation described in Sec. 2.2 shows that at $t = 0$ the diagonal matrix elements $h_{11}(0), h_{22}(0)$ of the effective Hamiltonian $H_{\|}(t)$ are equal, which means that masses of particles nad their antiparticles are equal at the instant of their creation. The property (60) occurs for $t > T_{as}^{X} > 0$, (here T_{as}^{X}

is calculated for the pairs of particles $X, \overline{X}$ discussed in the previous Section and it corresponnds to the time T_{as}, which was calculated for the neutral K system), which means (as it follows from Sec. 5.2) that if the baryon number B is not conserved and if CP symmetry is violated then the asymmetry between the numbers of unstable baryons and antibaryons can arise in a CPT invariant system at $t > T_{as}^{X} > 0$ even in the thermal equilibrium state of this system. This means that the effect described in Sec. 3 can replace the major part of models of type considered in [26] — [30] and it has major advantage over those models, namely it does not require CPT symmetry to be violated.

In this place the one more problem should be touched. Namely the quantum field theory provides us with the recipe of how to construct some charge and number operators. The baryon number operator $\hat{B}$ is used in some papers in order to prove that the third Sakharov condition for bryogenesis is necessary. Namely in these papers assuming (51) the equilibrium average of baryons, $< B >_T$, was calulated

$$\begin{aligned} < B >_T &= \mathrm{Tr}\,(e^{-\beta H}\hat{B}) \;=\; \mathrm{Tr}\,\Big(\Theta^{-1}\Theta(e^{-\beta H}\hat{B})\Big) \\ &\equiv \mathrm{Tr}\,\Big(\Theta(e^{-\beta H}\hat{B})\Theta^{-1}\Big) \;= - < B >_T . \end{aligned} \tag{106}$$

From this equation it follows that $< B >_T = 0$, which is considered in the large literature as the proof that there cannot be any generation of the net of baryon number in equilibrium. Such a conclusion is true, but only for stable baryons (generally, only for stable particles). This is because such operators as the baryon number operator (and in general any charge operator) are expressed by means of fields corresponding to the particles considered. There are no quantum fields which correspond to unstable particles. Therefore any charge operators for unstable particles do not exist. Simply, calculations leading to the relation (106) cannot be performed in the case of unstable particles. This means that the conclusions following from the relation (106) can not be extended to the case of unstable elementary quantum objects which are considered in this paper. Mathematics and logic give no reasons for which consequences of the relation (106) can be extrapolated to the case of unstable particles.

It seems that estimations (62), (64) should be also taken into account when one wants to interepret tests of quantum mechanics and CPT symmetry in the neutral kaon system [38, 39]. Indeed, the parameters used in [38] to

describe the deviations of quantum mechanics, or violations of CPT, are of similar order to (62), (64) [39, 40] . This means that the interpretation of CPT tests, or tests of modified quantum mechanics, based on the theory developed in [38, 39] may be incorretct. A similar conclusion seems to be right with reference to the theories describing effects of external fields on the neutral kaon system [41]. Also, the interpretation of tests of special relativity and of the equivalence principle [42] is based on the standard form, (22), (23) of the H_{LOY}. The order of the effects discussed in [42] can be compared to (62). So it seems to be obvious that the application of $H_{\|}$, (35), (50), (57) instead of H_{LOY}, when one considers theories developed in all these papers, can lead to conclusions which need not agree with those obtained in [38]—[42].

Finishing our considerations one ought to mention one more property of relations (58) and (60). Namely, if instead of (52), one has $[\mathcal{CP}, H] = 0$, then $h_{11} - h_{22} = 0$ in (58) and $M_{11} - M_{22} = 0$ in (60) for stable and unstable states $|\mathbf{1}>, |\mathbf{2}>$ [19].

Last of all it should be emphasized that the relations obtained in Sec. 3 and conclusions (58), (60) and others following from them are not a hypothesis. This is a rigorous mathematical result obtained within the use of basic assumptions of Quantum Mechanics.

A Appendix

Performing the integration by parts in (80) one finds

$$I_N^{\pm}(a) \equiv \frac{1}{3}\int_a^{\infty}(x^2-a^2)^{\frac{3}{2}}\frac{e^x}{(e^x \pm 1)^2}dx \tag{A.1}$$

$$= \frac{1}{3}\int_a^{\infty}(x^2-a^2)^{\frac{3}{2}}\frac{e^{-x}}{(1 \pm e^{-x})^2}dx, \tag{A.2}$$

where $a > 0$.

The function $I_{\Delta N}^{\pm}(z)$, (83), is defined as follows

$$I_{\Delta N}^{\pm}(z) \stackrel{\text{def}}{=} \left.\frac{\partial I_N^{\pm}(x)}{\partial x}\right|_{x=z}$$

$$= \quad -z\int_{z>0}^{\infty}(x^2-z^2)^{\frac{1}{2}}\frac{e^x}{(e^x\pm 1)^2}dx \tag{A.3}$$

$$= \quad -z\int_{z>0}^{\infty}(x^2-z^2)^{\frac{1}{2}}\frac{e^{-x}}{(1\pm e^{-x})^2}dx. \tag{A.4}$$

Now let us consider the integral

$$\mathcal{J}_l^{\pm}(z) \stackrel{\text{def}}{=} \int_{z>0}^{\infty}(x^2-z^2)^{\frac{l}{2}}\frac{e^{-x}}{(1\pm e^{-x})^2}dx. \tag{A.5}$$

This integral can be transformed into the following one

$$\mathcal{J}_l^{\pm}(z) \quad = \quad z^{l+1}\int_1^{\infty}(x^2-1)^{\frac{l}{2}}\frac{e^{-zx}}{(1\pm e^{-zx})^2}dx, \tag{A.6}$$

$$\equiv \quad z^{l+1}\sum_{n=1}^{\infty}(\mp 1)^n n\int_1^{\infty}(x^2-1)^{\frac{l}{2}}e^{-nzx}\,dx, \tag{A.7}$$

(where $z>0$) which can be intergrated term by term. This leads to the following result

$$\mathcal{J}_l^{\pm}(z) = \mp\frac{2^{\frac{l+1}{2}}}{\sqrt{\pi}}\,\Gamma(\tfrac{l+2}{2})\,a^{\frac{l+1}{2}}\sum_{n=1}^{\infty}(\mp 1)^n\,n^{-\frac{l-1}{2}}\,K_{\frac{l+1}{2}}(nz), \tag{A.8}$$

where $\Gamma(x)$ is the Gamma function and $K_n(x)$ is the modified Bessel function (see, eg, [43]),

$$K_{\frac{l+1}{2}}(z) = \left(\frac{z}{2}\right)^{\frac{l+1}{2}}\frac{\sqrt{\pi}}{\Gamma(\frac{l}{2}+1)}\int_1^{\infty}e^{-zt}(t^2-1)^{\frac{l}{2}}\,dt, \quad (z>0). \tag{A.9}$$

Thus the integrals $I_N^{\pm}(a)$, (80), (A.2) and $I_{\Delta N}^{\pm}(a)$, (83), (A.4), equal

$$\begin{aligned} I_N^{\pm}(a) &= \frac{1}{3}\mathcal{J}_3^{\pm}(a) \\ &= \mp a^2\sum_{n=1}^{\infty}\frac{(\mp 1)^n}{n}K_2(an), \qquad (A.10) \\ I_{\Delta N}^{\pm}(a) &= -a\mathcal{J}_1^{\pm}(a) \\ &= \pm a^2\sum_{n=1}^{\infty}(\mp 1)^n K_1(an). \qquad (A.11) \end{aligned}$$

The series representations as well as the asymptotic expansions of $K_1(z)$, $K_2(z)$ for $z \ll 1$ and for $z \gg 1$ can be found, e.g.,in [43]. Unfortunately the mentioned asymptotic expansions are useless when one tries to estimate the infinite series appearing in (A.10), (A.11). This can be achieved if to use the following more simple estimations. Namely, using relation (A.9) one can majorize the modified Bessel function, eg., as follows

$$K_{\frac{l+1}{2}}(z) \ < \ K^{up}_{\frac{l+1}{2}}(z) \stackrel{\text{def}}{=} \Big(\frac{z}{2}\Big)^{\frac{l+1}{2}} \frac{\sqrt{\pi}}{\Gamma(\frac{l}{2}+1)} \int_1^\infty e^{-zt} t^l \, dt, \quad (z>0). \tag{A.12}$$

The function $K^{up}_{\frac{l+1}{2}}(z)$ equals

$$K^{up}_{\frac{l+1}{2}}(z>0) = \Big(\frac{z}{2}\Big)^{\frac{l+1}{2}} \frac{\sqrt{\pi}}{\Gamma(\frac{l}{2}+1)} e^{-z} \Big\{\frac{1}{z} + \sum_{k=1}^{l} \frac{l(l-1)\cdot \ldots \cdot (l-k+1)}{z^{k+1}}\Big\}. \tag{A.13}$$

The simplest lower bound $K^{low}_{\frac{l+1}{2}}(z)$ for $K_{\frac{l+1}{2}}(z>0)$ can be found analogously. From (A.9) one finds that

$$K_{\frac{l+1}{2}}(z>0) \ > \ K^{low}_{\frac{l+1}{2}}(z>0),$$

where

$$\begin{aligned} K^{low}_{\frac{l+1}{2}}(z>0) \ &\stackrel{\text{def}}{=} \ \Big(\frac{z}{2}\Big)^{\frac{l+1}{2}} \frac{\sqrt{\pi}}{\Gamma(\frac{l}{2}+1)} \int_1^\infty e^{-zt} (t-1)^l \, dt & \text{(A.14)} \\ &= \ \frac{\sqrt{\pi}}{2^{\frac{l+1}{2}}} \frac{l!}{\Gamma(\frac{l}{2}+1)} \frac{e^{-z}}{z^{\frac{l+1}{2}}}. \ \ (z>0) & \text{(A.15)} \end{aligned}$$

From (A.13) one infers that functions $K^{up}_1(z)$ and $K^{up}_2(z)$, which majorize $K_1(z)$ and $K_2(z)$ appearing in formuale (A.11) and (A.10), are equal to

$$\begin{aligned} K^{up}_1(z>0) \ &= \ e^{-z}(1+\frac{1}{z}), & \text{(A.16)} \\ K^{up}_2(z>0) \ &= \ \frac{e^{-z}}{3}(z+3+\frac{6}{z}+\frac{6}{z^2}). & \text{(A.17)} \end{aligned}$$

These relations enable us to find

$$\begin{aligned} K^{up}_1(z>0)\big|_{z\ll 1} \ &\simeq \ \frac{1}{z}, & \text{(A.18)} \\ K^{up}_1(z)\big|_{z\gg 1} \ &\simeq \ e^{-z}, & \text{(A.19)} \end{aligned}$$

and

$$
\begin{aligned}
K_2^{up}(z>0)\big|_{z\ll 1} &\simeq \frac{2}{z^2}, && \text{(A.20)}\\
K_2^{up}(z)\big|_{z\gg 1} &\simeq \frac{1}{3}e^{-z}z. && \text{(A.21)}
\end{aligned}
$$

From (A.15) one obtains

$$
\begin{aligned}
K_1^{low}(z>0) &= \frac{e^{-z}}{z}, && \text{(A.22)}\\
K_2^{low}(z>0) &= 2\frac{e^{-z}}{z^2}, && \text{(A.23)}
\end{aligned}
$$

which means that

$$
\begin{aligned}
K_1^{low}(z>0)\big|_{z\ll 1} &= \frac{1}{z}, && \text{(A.24)}\\
K_2^{low}(z>0)_{\,z\ll 1} &= \frac{2}{z^2}. && \text{(A.25)}
\end{aligned}
$$

B Appendix

The answer for the question if the effect described in Sec. 3 is able to generate a such particle–antiparicle masses diference, which has significant contribution into the observed matter–antimatter asymmetry, can be found using the simplet estimations of the integrals $I_N^{\pm}(\frac{M}{T})$ and $I_{\Delta N}^{\pm}(\frac{M}{T})$. In order to do this the use of the best estimations for the modified Bessel functions appearing in (A.10), (A.11) is not necessary. The sufficient estimations can be obtained directly from (A.2) and (A.4). It is easily to find that

$$
-\,I_{\Delta N,min}^{\pm}(z>0)\;<\;-I_{\Delta N}^{\pm}(z>0)\;<\;-I_{\Delta N,mx}^{\pm}(z>0), \qquad \text{(B.1)}
$$

where

$$
\begin{aligned}
-\,I_{\Delta N,min}^{-}(z>0) &\overset{\text{def}}{=} z\int_{z>0}^{\infty}(x^2-z^2)^{\frac{1}{2}}\,e^{-x}\,dx \;\equiv\; z^2K_1(z), && \text{(B.2)}\\
-I_{\Delta N,mx}^{-}(z>0) &\overset{\text{def}}{=} \frac{z}{(1-e^{-z})^2}\int_{z>0}^{\infty}(x^2-z^2)^{\frac{1}{2}}\,e^{-x}\,dx\\
&\equiv \frac{z^2}{(1-e^{-z})^2}K_1(z), && \text{(B.3)}
\end{aligned}
$$

and

$$-\,I^{+}_{\Delta N,min}(z>0) \;\stackrel{\rm def}{=}\; \frac{z}{(1+e^{-z})^2}\int_{z>0}^{\infty}(x^2-z^2)^{\frac{1}{2}}\,e^{-x}\,dx$$

$$\equiv\; \frac{z^2}{(1+e^{-z})^2}K_1(z), \tag{B.4}$$

$$-I^{+}_{\Delta N,mx}(z>0) \;\stackrel{\rm def}{=}\; z\int_{z>0}^{\infty}(x^2-z^2)^{\frac{1}{2}}\,e^{-x}\,dx \;\equiv\; z^2K_1(z). \tag{B.5}$$

Treating integral (A.2) analogously yields

$$I^{\pm}_{N,min}(z>0) \;<\; I^{\pm}_{\Delta N}(z>0) \;<\; I^{\pm}_{\Delta N,mx}(z>0), \tag{B.6}$$

where

$$I^{-}_{N,min}(z>0) \;\stackrel{\rm def}{=}\; \frac{1}{3}\int_{z>0}^{\infty}(x^2-z^2)^{\frac{3}{2}}\,e^{-x}\,dx \;\equiv\; z^2K_2(z), \tag{B.7}$$

$$I^{-}_{N,mx}(z>0) \;\stackrel{\rm def}{=}\; \frac{1}{3(1-e^{-z})^2}\int_{z>0}^{\infty}(x^2-z^2)^{\frac{3}{2}}\,e^{-x}\,dx$$

$$\equiv\; \frac{z^2}{(1-e^{-z})^2}K_2(z), \tag{B.8}$$

and

$$I^{+}_{N,min}(z>0) \;\stackrel{\rm def}{=}\; \frac{1}{3(1+e^{-z})^2}\int_{z>0}^{\infty}(x^2-z^2)^{\frac{3}{2}}\,e^{-x}\,dx$$

$$\equiv\; \frac{z^2}{(1+e^{-z})^2}K_2(z), \tag{B.9}$$

$$I^{+}_{N,mx}(z>0) \;\stackrel{\rm def}{=}\; \frac{1}{3}\int_{z>0}^{\infty}(x^2-z^2)^{\frac{3}{2}}\,e^{-x}\,dx \;\equiv\; z^2K_2(z). \tag{B.10}$$

Analyzing the series representations for $K_1(z)$ and $K_2(z)$ one concludes that the leading terms for $z \ll 1$ [43] are

$$K_1(z) \;\simeq\; \frac{1}{z},\;\; (z \ll 1),$$

$$K_2(z) \;\simeq\; \frac{2}{z^2},\;\; (z \ll 1), \tag{B.11}$$

and that for $z \gg 1$ the leading term of the modified Bessel functions is [43]

$$K_n(z) \simeq \sqrt{\frac{\pi}{2z}}e^{-z},\;\; (z \gg 1,\; n=1,2,\ldots). \tag{B.12}$$

C Appendix

For completeness, one more estimation of the integral, $I_N^{\pm}(z)$, will be considered. Namely, taking into account that $x \geq a > 0$ one finds from (80) that the simplest upper bound of $I_N^{\pm}(z)$ for $z > 0$ can be chosen as follows

$$I_N^{\pm}(z) \; < \; \Upsilon^{\pm}(z) \stackrel{\mathrm{def}}{=} \int_z^{\infty} \frac{x^2}{e^x \pm 1}\, dx \; \leq \; \int_0^{\infty} \frac{x^2}{e^x \pm 1}\, dx. \tag{C.1}$$

This means that (see [43])

$$\Upsilon^{+}(z) \leq \frac{3}{4}\Gamma(3)\zeta(3) = \frac{3}{2}\zeta(3), \tag{C.2}$$

and

$$\Upsilon^{-}(z) \leq \Gamma(3)\zeta(3) = 2\zeta(3). \tag{C.3}$$

Here $\zeta(3)$ is the Riemann's Zeta function of 3. These estimations are valid for any $z > 0$ but in the literature are considered as approximate values of the integral (80) for relativistic particles, $z = \frac{m}{T} \gg 1$, only (see,eg., [24]).

D Appendix

If one considers, eg., the estimation (C.3) as the exact value of the integral $I_N^{-}(z)$ then one obtains the following expressions for $\eta^{-}_{x,min}$ and $\eta^{-}_{x,mx}$,

$$\eta^{-}_{x,mx} = \frac{\Delta m}{T}\frac{[-I^{-}_{\Delta N,mx}(z)]}{2\zeta(3)} = \frac{1}{2\zeta(3)}\frac{z^2}{(1-e^{-z})^2}K_1(z)\bigg|_{z=\frac{m}{T}}, \tag{D.1}$$

and

$$\eta^{-}_{x,min} = \frac{\Delta m}{T}\frac{[-I^{-}_{\Delta N,min}(z)]}{2\zeta(3)} = \frac{z^2 K_1(z)}{2\zeta(3)}\bigg|_{z=\frac{m}{T}}. \tag{D.2}$$

These relations and, for instance, (B.11) give

$$\eta^{-}_{x,mx} \simeq \frac{1}{2\zeta(3)}\frac{\Delta m}{m} \sim \frac{\Delta m}{m}, \quad (\tfrac{m}{T} \ll 1), \tag{D.3}$$

and

$$\eta^{-}_{x,min} \simeq \frac{1}{2\zeta(3)}\frac{\Delta m}{T}\frac{m}{T} \sim \frac{\Delta m}{T}\frac{m}{T}, \quad (\tfrac{m}{T} \gg 1). \tag{D.4}$$

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